THE ASTROPHYSICAL JOURNAL, 211:342–356, 1977 January 15

DIRAC COSMOLOGY

V. CANUTO
Institute for Space Studies, NASA, New York
AND
J. LODENQUAI
Department of Physics, University of the West Indies, Kingston, Jamaica
Received 1975 December 18

ABSTRACT

In this paper we study Dirac's suggestion that the large dimensionless numbers ($\sim 10^{40}$) encountered in physics owe their large sizes to their proportionality to the age of the Universe. After presenting in detail the foundations of the theory and comparing them with those of the big bang, we check the predictions of the theory against the 3 K background radiation and the (log N, log S)-relation for radio galaxies. We analyze the criticisms of Gamow and Teller to an early version of the theory regarding the time variation of the Sun's luminosity. We find that the present version of the theory is free from such difficulties.

Finally, we try to explain the unusually small value of $\dot{P}/P$ of the pulsar JP 1953 via the time variations of both G and the mass of the pulsar. Present upper limits on $\dot{G}/G$ make such an explanation a viable one. In no case is Dirac's cosmology found to contradict any well-established observational facts. Difficulties may, however, arise in the explanation of the 3 K background radiation and with the low-luminosity white dwarfs.

Subject headings: cosmic background radiation — cosmology

I. INTRODUCTION

A general belief among physicists is that dimensionless numbers, like the fine structure constant, derived from the combinations of fundamental physical constants will eventually emerge from some as yet nonexistent theory. Some of these dimensionless numbers resulting from combining atomic and astronomical constants are, however, exceedingly large to be expected to emerge from fundamental theories that usually involve numbers like 2, π, etc.

One such large number is the ratio of the electric to the gravitational force between an electron and a proton in a hydrogen atom,

$$\frac{\text{electric force}}{\text{gravitational force}} = \frac{e^2}{Gm_pm_e} \approx 2 \times 10^{39}\,. \tag{1}$$

Many years ago Dirac (1937) proposed an explanation for such large numbers. Dirac noticed that if the present age of the Universe is expressed in units of a fundamental or natural unit of time taken to be

$$\frac{e^2}{m_ec^3} \approx 10^{-23}\,\text{s}\,, \tag{2}$$

then the age of the Universe is

$$t_0 = \frac{H_0{}^{-1}}{e^2/m_ec^3} \approx 10^{40}\,, \tag{3}$$

where H_0 is the value of the Hubble constant today.[1] The surprising similarity of (1) and (3) prompted Dirac to propose that the above ratio is not in fact constant but is proportional to the age of the Universe t_0, i.e.,

$$\frac{e^2}{Gm_pm_e} \approx t_0\,. \tag{4}$$

Dirac assumed that the atomic constants such as e, m_e, m_p, and $\hbar$ are truly constants. In order for (4) to be valid, G must be inversely proportional to t; i.e.,

$$-\left(\frac{\dot{G}}{G}\right)_0 = \frac{1}{t_0} \approx 8.33 \times 10^{-11}\,\text{yr}^{-1}\,. \tag{5}$$

[1] In general the subscript 0 indicates the present value of the quantity to which it is affixed.

The latest experimental value for $(\dot{G}/G)_0$ according to Van Flandern (1975) is

$$-\left(\frac{\dot{G}}{G}\right) = (9 \pm 4) \times 10^{-11}\ \mathrm{yr}^{-1}, \tag{6}$$

which is definitely consistent with (5).

The unit of time (2) is a *natural* unit, independent of man-made units such as seconds and minutes. Physically $e^2/m_e c^3$ is the time taken for light to travel the classical electron radius. According to Dirac, the number $e^2/Gm_e m_p$ is large today simply because the Universe is now old.

The hypothesis that large dimensionless numbers should be proportional to the cosmic time t raised to some simple power is known as the large numbers hypothesis (LNH).

Since the LNH as it stands today has received relatively slight exposure (Dirac 1973*a*, *b*, *c*, 1974) and has frequently been misinterpreted, we intend to review it in some detail. We shall then examine some of its consequences on solar evolution, cosmology, and pulsar physics. Finally, we mention the alternative hypotheses of Dicke and Carter to explain the relations between the large numbers.

II. REVIEW OF THE LNH

a) The Large Numbers

Besides the relation (4) we may introduce three other relations from which many interesting large numbers relations follow.

If we calculate the ratio of the visible radius of the Universe defined as (c/H_0), to the classical electron radius $e^2/m_e c^2$, we obtain

$$\frac{cH_0^{-1}}{e^2/m_e c^2} \equiv \frac{R_0}{e^2/m_e c^2} \approx 10^{40} \approx t_0\,. \tag{7}$$

According to the LNH we must have, at any time t,

$$\frac{m_e c^3}{e^2 H} \approx t\,. \tag{8}$$

Next, if the estimated mass of all the matter in the Universe ($\rho \approx 10^{-31}\ \mathrm{g\ cm^{-3}}$) is divided by the proton mass m_p, we derive an estimate of the total number of nucleons in the Universe at the present time. This number N_0 turns out to be $\approx 10^{78}$, which is remarkably close to the square of (3). According to the LNH, at any time t the total number of nucleons N in the Universe must then have been proportional to t^2, i.e., to the age of the Universe at that time t,

$$N \approx t^2\,. \tag{9}$$

This is perhaps the most important consequence of Dirac's LNH, since it implies that matter in the Universe increases with time. We shall discuss this result in more detail below.

Finally, if we take the ratio of the classical electron radius r_e, to the present Planck length l_P defined as

$$l_\mathrm{P} = (\hbar G/c^3)_0^{1/2} \approx 10^{-33}\ \mathrm{cm}\,, \tag{10}$$

we obtain

$$\frac{e^2}{(m_e^2 \hbar c G_0)^{1/2}} \approx 10^{20} \approx t_0^{1/2}\,, \tag{11}$$

and so quite generally, at any time t,

$$\frac{e^2}{(m_e^2 \hbar G c)^{1/2}} \approx t^{1/2}\,. \tag{12}$$

The dimensionless constants such as $e^2/\hbar c$, m_p/m_e must be considered of the order of unity, i.e., $\approx t^0$ in the LNH.

Relations (4), (8), (9), and (12) can be manipulated to yield interesting relations. For example, if we multiply (8) by (12) and use (9) to eliminate t, we obtain

$$c/H \equiv R \approx N^{3/4}(\hbar G/c^3)^{1/2}\,, \tag{13}$$

which expresses the radius of the Universe in terms of the total number of nucleons and other fundamental physical parameters. Next, if we combine (8) with (9), we get

$$m_e c^2 = \frac{e^2 N^{1/2}}{c/H} = \frac{e^2 N^{1/2}}{R} \,. \tag{14}$$

This relation has been arrived at by Narlikar (1974), who assumed a departure from strict charge neutrality in the Universe; i.e., he assumed that the charge balance is a statistical effect. Hence equation (14) would be interpreted as saying that the electrostatic energy of charge fluctuation in the Universe is equal to the rest mass energy of the electron. If we now eliminate the radius of the Universe between equations (13) and (14), we obtain a relation for the mass of the Universe M_u, i.e.,

$$M_u = m_p N \approx (e^2/m_e c^2)^4 (c^3/\hbar G)^2 m_p \,. \tag{15}$$

Finally, if we combine (4) and (8), we obtain the relation

$$GM_u H/c^3 \approx 1 \,, \tag{16}$$

which is usually regarded as expressing Mach's principle (Sciama 1953):

$$mc^2 = mGM_u/R \,. \tag{17}$$

b) *Continuous Creation*

We have seen that (9) implies that the amount of matter in the Universe must increase as t^2, i.e., matter must be continuously created. According to Dirac (1974) there are two processes whereby matter is created: (*a*) Matter is created uniformly throughout space and therefore mainly in intergalactic space. This type of creation is called *additive creation*. (*b*) Matter is created where other matter already exists and in proportion to the amount already existing. The created matter is assumed to be of the same kind as that already existing. This type of creation is called *multiplicative creation*. This process would require a fractional increase in the mass of a rock in the amount of $\dot{M}/M \approx 2/t \approx 2 \times 10^{-10}$ per year at present. This is unfortunately too small to be observed in the laboratory. However, during the course of geological times a piece of rock would grow in size and mass, and that could lead to some observational features. Similarly, the mass of a star would increase as t^2. On the other hand, in additive creation, matter is created uniformly throughout space and therefore mainly in intergalactic space. The mass of a star, for example, would change negligibly compared with that in multiplicative creation, and so we would have a theory in which practically only G varies.

From the investigations of Pochoda and Schwarzschild (1964) and Gamow (1967), the LNH with additive creation was shown to contradict several accepted facts. The investigation of Teller (1948) was also unfavorable to additive creation; but because the assumed age of the Universe was a factor of 3 smaller than the presently accepted value, Teller's conclusion is no longer valid although his method is still valid.

Pochoda and Schwarzschild studied numerically the evolution of the Sun with only the gravitational constant G varying as t^{-1}. They concluded that the Sun would have evolved so quickly that it would now be in the red giant phase, contrary to observation. The same conclusion was reached by Gamow, whose method based on the homology transformation of the equations of stellar structure is especially transparent. The above investigations were decisively not in favor of the LNH with additive creation.

In the last two years, Dirac (1973*a*, *b*, *c*, 1974) has prepared an extended version of the LNH in which multiplicative creation is advocated. This new version of the theory leads to the introduction of two types of metrics, the atomic and Einstein metrics, which we now discuss.

c) *The Atomic and Einstein Metrics*

Since Dirac did not want the LNH to contradict Einstein's general theory of relativity (which is based on a constant G and conservation of mass-energy), Dirac introduced two metrics, the atomic metric and the Einstein or mechanical metric.

The atomic metric is measured by atomic instruments and therefore results from experimental measurements. On the other hand, the Einstein field equations are written in the Einstein metric with constant G and mass-energy conservation. As Dirac has repeatedly stressed, Einstein units should be used when one deals with mechanical laws like the motion of stars, planets, etc., whereas the atomic units are to be employed when one deals with atomic or nuclear phenomena. Alternatively, Einstein units (E) can be characterized by saying that

$$G_{\rm E} \sim \text{const.} \,, \qquad M_{\rm E} \sim \text{const.} \,, \tag{18}$$

whereas in atomic units we must have

$$e,\, \hbar,\, m \sim \text{const.} \tag{19}$$

The question now arises: How do G and M vary in atomic units, and how do e, $\hbar$, and m vary in Einstein units?

In order to answer these questions, let us consider equations (4) and (9). From (4) we have that

$$G_{\mathrm{E}} \sim \text{const.}\,, \qquad \left(\frac{e^2}{m^2}\right)_{\mathrm{E}} \sim t \quad \text{(Einstein units)}\,; \tag{20}$$

$$e, \hbar, m \sim \text{const.}\,, \qquad G \sim t^{-1} \quad \text{(atomic units)}\,. \tag{21}$$

From the definition of total mass of an object

$$M \approx mN$$

and equation (9) we deduce, because of (18),

$$m_{\mathrm{E}} \sim t^{-2}\,; \tag{22}$$

i.e., the mass of every elementary particle decreases like (cosmic time)$^{-2}$, when measured in Einstein units. From (20) we finally obtain

$$e_{\mathrm{E}}^2 \sim t^{-3}\,, \qquad \hbar_{\mathrm{E}} \sim t^{-3} \tag{23}$$

if $e^2/\hbar c$ is constant, as it appears to be. In fact, the work of Bahcall and Schmidt (1967) indicated that the value of α when determined from radio galaxies of large redshifts (and therefore of different epochs) does not vary appreciably from its values at $z = 0$, i.e., today. In particular,

$$\alpha(z = 1.95) = (0.97 \pm 0.005)\alpha(z = 0)\,.$$

Equation (18) supplemented by (22) and (23) defines the Einstein units, whereas equation (19) together with (21) specifies the atomic units.

The next question concerns the variation of lengths (both distances as well as wavelengths) in the two systems of units. To that end, let us consider the motion of a given particle about a central massive object of mass M in the Newtonian approximation. The basic equation

$$GM = v^2 r\,, \tag{24}$$

where v is the orbital velocity and r the radius of the orbit, expresses a balance of forces and must evidently hold in either system of units. By definition, in Einstein units G_{E} and M_{E} are constant. Since the velocity is essentially a dimensionless quantity, being a fraction of the velocity of light c, we must have

$$r_{\mathrm{E}} \sim \text{const.} \tag{25}$$

In Einstein units, the distance between two galaxies does not change with time, and therefore there is no expansion and consequently no redshift. On the contrary, in atomic units equation (24) implies that

$$r \sim t\,. \tag{26}$$

In general, the relation between the atomic and Einstein metrics is given by

$$ds = t ds_{\mathrm{E}}\,. \tag{27}$$

In atomic units there is expansion, and therefore we can expect the usual interpretation of many of the cosmological well-known facts. We shall discuss them in detail later.

Table 1 summarizes the relations of various parameters with the atomic time t in both atomic and Einstein metrics. From this table the time-dependence of various combinations of the listed parameters in either unit can readily be determined. For example, the Bohr radius $a_{\mathrm{B}} = \hbar^2/m_e c^2$ is constant in atomic units but varies as t^{-1} when measured with the Einstein unit of length. The atomic unit of time $e^2/m_e c^3$ now varies as t^{-1} in the Einstein metric.

III. COSMOLOGICAL IMPLICATIONS OF THE LNH

a) Open Universe

An immediate cosmological consequence of the LNH is that the Universe cannot be oscillating. If this were so, then there would exist a time $t_m \geqslant t_0$ at which the Universe reaches its maximum radius, say R_m. From Einstein's equations it is known that t_m is given by

$$t_m = \frac{\pi q}{H(2q - 1)^{3/2}}\,.$$

TABLE 1

DEPENDENCE OF VARIOUS PARAMETERS WITH TIME t IN BOTH ATOMIC AND EINSTEIN METRICS

Parameter	Atomic Units	Einstein Units
v (velocity)	t^0	t^0
e (charge)	t^0	$t^{-3/2}$
m (mass)	t^0	t^{-2}
$\hbar$ (Planck's constant)	t^0	t^{-3}
G (gravitational constant)	t^{-1}	t^0
M (bulk mass)	t^2	t^0
r (distance)	t	t^0
λ (wavelength)	t	t^0
($\hbar^2/me^2$), Bohr radius	t^0	t^{-1}

Since t_m depends upon H and q, whose values in turn depend on the epoch at which they are evaluated, one could imagine that t_m depends on the time at which q and H are evaluated. This would be very strange indeed since it would mean that the maximum size of the Universe would change as the Universe gets older. It is, however, easy to prove that

$$\frac{d}{dt} t_m = 0\,;$$

i.e., t_m is a true large number completely unrelated to the present epoch. It is therefore physically (not arithmetically!) impossible to write it as t^n, since it is not related to the age of the Universe at any epoch.

Since the LNH allows only for those large numbers that *can* on physical grounds be expected to depend on t, it is clear that the model with a t_m must be excluded; i.e., the Universe cannot be oscillating.

b) *Law of Expansion*

The LNH also results in a law for the expansion of the Universe. Let the distance between two galaxies when measured in atomic units be $kf(t)$. The quantity k depends on the pair of galaxies chosen, whereas $f(t)$ is assumed to be the same for all pairs of galaxies for a uniform universe. Asymptotically, $f \sim t^n$, so that we can write

$$R = kt^n\,. \tag{28}$$

The velocity of recession $\dot{R}$ is knt^{n-1}. If $n > 1$, we then have an accelerating expansion and the velocities of distant galaxies will eventually exceed c, which is impossible.

If $n < 1$, corresponding to a decelerating expansion, then there existed a time t_* in the past when $\dot{R} \sim c$. Since the present recession velocity of a high-redshift galaxy is about $10^{-3}c$, we have

$$\dot{R}_0 \approx 10^{-3}c = knt_0^{\,n-1}\,,$$

and so the time t_* given by

$$\dot{R}_* = c = knt_*^{\,n-1}$$

can be expressed in terms of t_0 as

$$t_* = t_0 \times 10^{3/(n-1)}\,.$$

For $n = \frac{3}{4}$, say, we have

$$t_* \approx 10^{-12} t_0 \approx 10^{28}\,.$$

This is again a large number unrelated to the present age t_0 and therefore not expressible in terms of it. The LNH leaves no room for such numbers, and the possibility of a universe with significant acceleration must consequently be ruled out.

In conclusion, both $n > 1$ and $n < 1$ are inadmissible, and the only remaining possibility is $n = 1$, i.e., a uniform expansion:

$$R = kt\,. \tag{29}$$

This is in agreement with the general behavior of lengths determined before (Table 1). In atomic units, distances increase proportionally to the cosmic time.

Evidently in Einstein units any length and in particular the radius of the Universe do not change with time, i.e.,

$$R_{\mathrm{E}} \sim \text{const.} \tag{30}$$

There is only one cosmological model that allows for a constant radius: the original Einstein static universe.

Unless we allow for $3p + \epsilon \leq 0$, a static universe can be arrived at only after one introduces a cosmological constant, Λ_{E}. In Einstein units the metric ds_{E} has the form

$$ds_{\mathrm{E}}^2 = c^2 dt_{\mathrm{E}}^2 - \left(\frac{dr_{\mathrm{E}}^2}{1 - r_{\mathrm{E}}^2/R_{\mathrm{E}}^2} + r_{\mathrm{E}}^2 d\Omega^2\right), \tag{31}$$

where R_{E} is a constant related to Λ_{E} and to the density ρ by

$$\Lambda_{\mathrm{E}} = R_{\mathrm{E}}^{-2}, \qquad \rho_{\mathrm{E}} = \left(\frac{\Lambda c^2}{4\pi G}\right)_{\mathrm{E}}. \tag{32}$$

The historical reason behind the rejection of Einstein's model was the discovery by Hubble that the redshift was proportional to the distance of the galaxy itself, thus giving rise to the cosmological interpretation of the redshift, i.e.,

$$z + 1 = \frac{R(t_0)}{R(t)}. \tag{33}$$

In a static universe z is clearly zero and the spectral lines should not be shifted. However in Dirac's theory, as we shall see, the redshift is caused by a different mechanism and therefore the static universe cannot be rejected.

In atomic units (29) is valid and so we can define a Hubble constant

$$H = \dot{R}/R \sim t^{-1} \tag{34a}$$

and the acceleration parameter

$$q = -\ddot{R}R/\dot{R}^2 = 0. \tag{34b}$$

The history of q in the last 15 years has been recounted several times, and there is no point in repeating it. The cosmological test par excellence, the (m, z) relation marshaled for many years by Sandage and collaborators, until very recently gave results that did not preclude the possibility of a closed universe (Sandage 1968, 1972, 1974).

In ordinary parlance the curvature k is related to q_0 through

$$\frac{k}{R_0^2} = 2q_0 - 1 = \frac{\rho}{\rho_c} - 1, \qquad \rho_c = \frac{3H^2}{8\pi G}; \tag{35}$$

and so long as $q_0 \geq \frac{1}{2}$, $k \geq 0$ and the Universe is open.

Strictly speaking, the form of the curve m versus z cannot differentiate between two q's until one reaches at least a point where

$$\log cz \approx 4.8\text{–}5.0 \tag{36}$$

where the first QSO appears. In fact, the first-ranked galaxies used by Sandage could be fitted by the straight line

$$m = 5 \log cz - 6.46, \tag{37}$$

which is strictly valid for $q_0 = 1$. The same argument is true for isophotal angular diameters.

Recently, however, Sandage and Tammann (1975) have decided to employ a different technique to determine q_0. If one could determine H independently of q, i.e., using nearby objects for which the effects of curvature are not significant, then one could employ the well-known formula

$$t = H_0^{-1} f(q_0) \tag{38}$$

giving the age of the Universe once H_0 and q_0 are known. The function $f(q_0)$ is unity at $q_0 = 0$ and decreases monotonically as q_0 increases. If a value of

$$H_0 = 55 \pm 3 \text{ km s}^{-1} \text{ Mpc}^{-1} \tag{39}$$

is used, it turns out that q_0 must be less than 0.5 in order to provide an age at least larger than that of globular clusters. Even though the quoted value of q_0 has still a large error, it is definitely indicative of an open universe. This is as of 1975 March. It is important to notice that if nucleosynthesis is indeed primordial, the amount of deuterium formed is a very sensitive function of q_0. The observational abundance of deuterium definitely indicates a value of q_0 much less than 1, i.e., an open universe (Gott *et al.* 1975). A value of $q_0 = 0$ is not in contradiction with any presently known observational data. Evidently the determination of q_0 from observational data is a slippery endeavor in that the theoretical analysis can become arbitrarily complex if one extends the theory to include the cosmological constant. One has the distinct feeling that somehow the theory as it stands today is not perfect, and there is too much freedom in the choice of parameters. Dirac's cosmology can be regarded as a way to close that gap by adopting the LNH. As we have seen, the value of q_0 is automatically determined to be zero, and so is the type of universe we must adopt: one that is nonaccelerating.

To make the presentation complete, we must now indicate how redshifts originate in Dirac's cosmology.

c) Redshift

Because the description of the Universe is different in the two metrics, we get different explanations for the redshifts of galaxies.

In Einstein units, the radius of the Universe is constant, so no redshift is observable if time is measured with Einstein clocks. However, all physical clocks are basically atomic clocks; and if Δt is a unit of time in the atomic metric, the corresponding unit in the Einstein metric is, according to equation (27),

$$(\Delta t)_{\mathrm{E}} = \Delta t/t \sim t^{-1}\,, \tag{40}$$

which decreases with time. Hence "atomic clocks" are speeding up with respect to "Einstein clocks." At two different epochs t_e and t_0 ($t_0 > t_e$) in atomic units, the corresponding time units in the Einstein metric can be written as

$$(\Delta t_{\mathrm{E}})_0 = (\Delta t_{\mathrm{E}})_e(t_e/t_0)\,. \tag{41}$$

Suppose now that a light signal characterized by a wavelength λ_e was emitted by a galaxy when the age of the universe was t_e. When the signal arrives at the measuring apparatus, the universe has an older age, say t_0, and the wavelength a value λ_0.

From (41) it follows that at any time t

$$\lambda(t) = \lambda_e\left(\frac{t}{t_e}\right) \tag{42}$$

so that the redshift becomes

$$z = \frac{\lambda_R - \lambda_L}{\lambda_L} = \frac{\lambda_R - \lambda_e}{\lambda_e} = \frac{\lambda(t_0) - \lambda_e}{\lambda_e} = \frac{t_0}{t_e} - 1 \tag{43}$$

on the grounds that λ_e(emission) $= \lambda_L$(Lab.), λ_R(Received) $= \lambda(t_0)$. This relation is evidently consistent with the general definition (33) together with (29).

d) (log N, log S)-*Relation*

We have seen that equation (9) requires the continuous creation of matter which must therefore include photons. Because of the continuous creation of photons, the apparent luminosities of distant galaxies will increase. Consider for simplicity a beam of monochromatic light that originated at a galaxy at time t_e. The energy of this beam in Einstein units is given by

$$E_{\mathrm{E}} = (h\nu N_\gamma)_{\mathrm{E}}\,, \tag{44}$$

where N_γ is the number of photons in the beam and ν the frequency. The wavelength will depend in general on the time of emission but will subsequently remain constant on the journey to the Earth in accordance with (25). However, since $\hbar_{\mathrm{E}} \sim t^{-3}$, while E_{E} is constant, we must have $(N_\gamma)_{\mathrm{E}} \sim t^3$; i.e., if the beam arrives at the Earth at time t_0, the fractional increase in the number of photons will be $(t_0/t_e)^3 = (1 + z)^3$, on the basis of (43). Since N_γ is a pure number, this relation must hold in atomic units, so quite generally the number of photons of a particular frequency must increase as

$$N_\gamma \sim t^3\,. \tag{45}$$

This spontaneous creation of photons in the Dirac LNH will affect many cosmological relations and in particular the (log N, log S)-relation for radio galaxies. Suppose $N(\geq S)$ is the total number of sources with flux $\geq S$. Then, all these sources will be contained in the sphere of radius r_s given by (Longair 1971)

$$r_s = \left[\frac{(1 + z_m)^3 L}{4\pi S}\right]^{1/2}, \tag{46}$$

where L is the absolute luminosity of a source and z_m the redshift of the source whose flux density just equals S. Then

$$N(\geq S) = \frac{4\pi}{3}\sigma r_s^3 = \frac{4\pi}{3}\sigma\left(\frac{L}{4\pi}\right)^{3/2} S^{-3/2}(1 + z_m)^{9/2}, \tag{47}$$

where σ is the density of sources. The redshift $z_m = z_m(S)$ is a decreasing function of S. This extra factor of $(1 + z_m)^{9/2}$, over the usual result, steepens the (log N, log S)-curve, in closer agreement with the experimental results. Of course, evolutionary corrections are not included and must be taken into account.

e) The Cosmic Background Radiation

The most commonly quoted argument in favor of the existence of a big-bang model is the existence of the 3 K background radiation, a relic characterizing the physical situation when the Universe had cooled to about 4000 K, i.e., when matter and radiation decoupled. How does this phenomenon fit into Dirac's theory? By using $T = 3$ K, we can construct a dimensionless number, namely,

$$m_e c^2/kT_0 = 10^{10} \approx (10^{40})^{1/4}. \tag{48}$$

If we identify this number with $t_0^{1/4}$, then according to the LNH we deduce that the temperature should decrease as

$$T \sim t^{-1/4}, \tag{49}$$

i.e., a much slower dependence than the ordinary

$$T \sim t^{-1}. \tag{50}$$

Since for blackbody radiation we have

$$\rho = E/V \sim T^4, \qquad N_\gamma/V \approx T^3, \tag{51}$$

it follows that

$$\rho \sim t^{-1}, \qquad N_\gamma \sim T^3\ V \sim t^{9/4}, \tag{52}$$

since in atomic units lengths increase with t. The number of photons increases with the 9/4 power of t, slower than the t^3 dependence we have found for a monochromatic beam. We can actually recover (52) from another observational fact. It is known that the present photon and matter densities are (Longair 1971)

$$N_\gamma/V \approx 400 \text{ photons cm}^{-3},$$
$$N_n/V \approx 10^{-7} \text{ nucleons cm}^{-3}; \tag{53}$$

i.e.,

$$N_\gamma/N_n \sim 10^{10} \sim t^{1/4}. \tag{54}$$

Since $N_n \sim t^2$, equation (52) is recovered. During the previous derivation we have used for m the mass of the electron. Had we used the mass of the nucleon, the results would have changed to

$$T \sim t^{-1/3}, \qquad \rho \sim t^{-4/3}, \qquad N_\gamma \sim t^2. \tag{55}$$

If so, the ratio N_γ/N_n would be independent of t since both N_γ and N_n increase like t^2. We would therefore have a large (experimental) dimensionless number which does not depend on the epoch and therefore cannot fit into the theory. On the other hand, the ratio N_γ/N_n is an experimental fact *only* if we regard the radiation as blackbody with $T = 3$ K, since this is the way one derives the first of (53). Since the present evidence is heavily in favor of a blackbody form, clearly N_γ must increase with t more rapidly than N_n does. Since the dependence of N_n is one of the basic points of the theory, its t^2 dependence cannot be tampered with and consequently we reach the conclusion that

$$N_\gamma \sim t^\alpha, \qquad \alpha > 2. \tag{56}$$

If so, then

$$T \sim t^{-(1-\alpha/3)}. \tag{57}$$

Since from (48) it is clear that T certainly decreases with t, we conclude that

$$2 < \alpha < 3. \tag{58}$$

The temperature decrease in time is gentler than in big-bang cosmology. The only possible way out is by saying that the present blackbody radiation originated not in the primeval fireball, but rather from some intergalactic ionized hydrogen. This is clearly a difficulty, since significant intergalactic material has not been found. It is not clear how much of such material is needed in Dirac's theory, but certainly it cannot be in excess of a critical value above which we would not be able to see distant objects like QSOs.

At the present moment a full theory has not yet been developed, and one cannot say how the curve should be modified. It is, however, clear that this point can constitute one of the major tests of the entire theory.

IV. ASTRONOMICAL IMPLICATIONS OF THE LNH

In this section we shall examine the original ideas of Teller (1948) and Gamow (1967) in connection with the present version of the LNH in order to test its consistency. These authors' works were based on the old version of the LNH with additive creation where only $G \sim t^{-1}$. Following this, we shall investigate the effects of the LNH on the period of pulsars (which can be measured with astonishing accuracy) to find out if the theory imposes any restrictions that might be detected.

a) Teller's Method

Teller showed that if $G \sim t^{-1}$, then about 4×10^8 years ago the luminosity of the Sun would have been so high that the temperature of the Earth would have exceeded the boiling point of water, thus precluding the existence of higher forms of life—contrary to paleontological evidence. Teller showed that the luminosity L of a star like the Sun is roughly proportional to G^7M^5 (cf. eqs. [62]), where M is the mass of the star. The temperature at the Earth's surface would be proportional to $(L/R^2)^{1/4}$, where R is the radius of the Earth's orbit. From (24), $R \sim (GM)^{-1}$, so T would vary as

$$T \sim G^{9/4}M^{7/4}\,. \tag{59}$$

Now if $G \sim t^{-1}$, then

$$T = T_0(t_0/t)^{9/4}\,, \tag{60}$$

where T is the Earth's temperature at cosmic time t and T_0 the temperature at cosmic time t_0 which we take to be the present. The above relation shows that T is a sensitive function of t, and this led Teller to the conclusion we mentioned above. However, as already stated, the value of t_0 chosen by Teller was about one-third the presently accepted value. This would push the "danger point" back about a billion years, for which paleontological data are scarce.

In the present version of the LNH, $G \sim t^{-1}$, $M \sim t^2$, and $R \sim t$, so

$$T \approx (L/R^2)^{1/4} \sim (G^7M^5/R^2)^{1/4} \approx T_0(t/t_0)^{1/4}\,. \tag{61}$$

We see that the qualitative effect of (61) is *opposite* to Teller's and furthermore T is a slowly varying function of time. In fact, 4×10^8 years ago the average surface temperature of the Earth was only 0.99 the present value. As Dirac has pointed out, the increase in temperature due to the increased radioactivity of the Earth in the past could easily have masked the effect of the LNH.

b) Gamow's Method

Gamow (1967) carried out an analytical study of the Sun's evolution based on the homology transformation of the equations of stellar structure. He concluded that if $G \sim t^{-1}$ only, then the luminosity of the Sun in the past would have been so high that all the nuclear fuel would have been used up by now and the Sun would now be in its red giant phase. A similar conclusion was reached by Pochoda and Schwarzschild (1964), who carried out a numerical investigation.

The idea of the method of homology transformation is this. If the gravitational constant G is changed by a factor g and the mass of the star is changed by a factor q, then from the full equations of stellar structure, the luminosity L will change by a factor l given by

$$l = g^{\alpha}q^{\beta}\,, \qquad \alpha = \frac{14n+45}{2n+5}\,, \qquad \beta = \frac{10n+31}{2n+5}\,, \tag{62}$$

provided that all remaining quantities are held fixed. In (62) n is the exponent of the temperature that results in expressing the equation of energy production in the star as

$$\frac{dL_r}{dr} = 4\pi r^2 c_1 c_2 \rho_r{}^2 \alpha T_r{}^n\,, \tag{63}$$

where L_r, ρ_r, and T_r are the luminosity, density, and temperature, respectively, at a distance r from the star's center, c_1 and c_2 are the concentrations of the two kinds of nuclear species participating in the reaction, and α and n depend on the particular reactions taking place and must therefore be calculated from nuclear physics. For stars with masses less than or close to that of the Sun, the p-p reaction dominates and $n \approx 4$. For more massive stars, the CN cycle dominates and $n \approx 17$. Teller's result corresponds to $n = \infty$.

Gamow applied these results to the Sun ($n = 4$), assuming that only G varies, i.e., $g \equiv G/G_0 = (t_0/t)$ and so

$$\frac{L}{L_0} = \left(\frac{G}{G_0}\right)^{7.8} = \left(\frac{t_0}{t}\right)^{7.8} .$$

The total amount of energy emitted between times t_1 and t_0 (the present) is given by

$$E_{10} = \int_{t_1}^{t_0} L(t)dt = L_0 t_0^{7.8} \int_{t_1}^{t_0} t^{-7.8} dt = \frac{1}{6.8} L_0 t_0 \left[\left(\frac{t_0}{t_1}\right)^{6.8} - 1\right] . \tag{64}$$

This amount of energy must not exceed the total amount of nuclear energy ϵ_0 available in the Sun's convective core for the H $\rightarrow$ He reaction, which Gamow estimates to be $\sim 0.6 M_0 \epsilon / m_p \approx 7.2 \times 10^{50}$ ergs. M_0 is the present solar mass, and $\epsilon \sim 10^{-5}$ ergs is the energy produced per proton in the H $\rightarrow$ He reaction. So we must have the inequality

$$\frac{1}{6.8} L_0 t_0 [(t_0/t_1)^{6.8} - 1] \leq \epsilon_0 ;$$

i.e.,

$$\frac{t_0}{t_1} \leq \left[\frac{5.4\ 10^{51}}{L_0 t_0} + 1\right]^{1/6.8} \equiv 1.28$$

if we take $L_0 \approx 4 \times 10^{33}$ ergs s^{-1} and $t_0 = 1.2 \times 10^{10}$ yr. This relation implies that the age of the Sun, $t_0 - t_1$, must satisfy the inequality

$$t_0 - t_1 = t_0(1 - t_1/t_0) > t_0(1 - 0.8) = 2.4 \times 10^9 \text{ yr} . \tag{65}$$

This period is much shorter than the age of the solar system (4 to 5 billion years) and therefore presumably that of the Sun. The conclusion is that if $G \sim t^{-1}$, the Sun would not be a main-sequence star today.

In the present case where $G \sim t^{-1}$ and $M \sim t^2$ we have

$$L/L_0 = (t_0/t)^{7.76} (t/t_0)^{10.92} = (t/t_0)^{3.15} ,$$

so the average luminosity of the Sun in the past must have been less than the present value L_0. Now, since the theory of stellar evolution shows that the Sun can easily exist up to the present time as a main-sequence star with a constant luminosity equal to the present value L_0, it can surely do so with an average luminosity that is less than the present value. In fact, if we apply Gamow's argument, we find that the following inequality must be satisfied:

$$\frac{1}{4.15} L_0 t_0 [1 - (t_1/t_0)^{4.15}] < \epsilon_0$$

or

$$t_1/t_0 > -1.0 ,$$

which is always satisfied. (See also Chin and Stothers 1975 for numerical results of some solar parameters in the LNH.)

The fact that $L \sim t^{3.5}$ in the present case means that the time spent on the main sequence by the Sun (or any other star) will be different from that expected from a more or less constant luminosity. If t_1 is the cosmic time of birth for the Sun and t_2 is the cosmic time of departure from the main sequence, then

$$\int_{t_1}^{t_2} L(t)dt = L_0 t_0^{-3.15} \int_{t_1}^{t_2} t^{3.15} dt = (L_0 t_0^{-3.15}/4.15)(t_2^{4.15} - t_1^{4.15}) = \epsilon_0 (t_2/t_0)^2 . \tag{66}$$

We have made the simplifying assumption that the available amount of nuclear fuel goes like t^2 as shown on the right of equation (66). Equation (66) can be solved numerically for $t_{\text{ms}} \equiv t_2 - t_1$, the total time the Sun spends on the main sequence. The result is

$$t_{\text{ms}} \approx 10^{10} \text{ yr} . \tag{67}$$

If the luminosity is constant at the present value of L_0, then the corresponding time on the main sequence is $\epsilon_0/L_0 \approx 5.8 \times 10^9$ yr. So the life expectancy of the Sun is increased by a factor ~ 1.8 in the LNH.

c) *Pulsar Period*

Shortly after the discovery of pulsars, Counselman and Shapiro (1968) proposed that the effects of a varying gravitational constant could be tested by measuring the time variation of the period of the pulses. This variation would depend on the particular model of the pulsar. If we accept the present model of a pulsar as an obliquely rotating magnetized neutron star, we can examine the effects on the period due to a varying G and varying mass M. Variations in G and M will cause a corresponding variation in the radius R of the star and hence a variation in its moment of inertia which in turn will affect the period of the pulses.

The discovery that the pulsar JP 1953 has an anomalously small rate of change of period $\sim 10^{-18}$ s s^{-1} (Richards, Rankin, and Zeissig 1974) instead of the typical value of $\sim 10^{-15}$ s s^{-1}, has aroused speculation that this extremely small slowing down rate could be the sole result of a varying gravitational constant. Before examining the effects of the LNH on the pulsar periods, we shall review the canonical theory of pulsar rotational periods in order to introduce the relevant notation and parameters.

In the usual model of a pulsar, an obliquely magnetized neutron star is losing its kinetic energy of rotation via the emission of magnetic dipole radiation:

$$\frac{d}{dt}(\tfrac{1}{2}I_0\Omega^2) = -\frac{\lambda}{2}\Omega^4 . \tag{68}$$

Here I_0 is the (constant) moment of inertia of the neutron star, Ω the rotational angular velocity, and $\lambda \equiv 4/3c^3 M_\perp^2$ is the component of the magnetic dipole moment perpendicular to the rotation axis of the star. Equation (68) neglects the effects of gravitational radiation which dominate only a short time (~ 100 years) after the formation of the pulsar. Equation (68) can be integrated to give

$$\Omega(t) = \frac{\Omega_1}{[1 + (\Omega_1^2\lambda/I_0)(t - t_1)]^{1/2}}, \tag{69}$$

where Ω_1 is the angular velocity at the initial time t_1. We can express (69) in terms of the present angular velocity Ω_0 and present age of the pulsar $\tau_0 = (t_0 - t_1)$, by putting $t = t_0$ in equation (69) to get

$$\Omega_1 = \Omega_0(1 + \tau_0/\tau_p)^{1/2}, \tag{70}$$

where

$$\tau_p = 3c^3 I_0/4M_\perp^2\Omega_0^2 = I_0/\lambda\Omega_0^2 . \tag{71}$$

If we substitute (70) in (69), we get

$$\Omega(t) = \frac{\Omega_0}{[1 + (\tau - \tau_0)/\tau_p]^{1/2}}, \tag{72}$$

where τ is the age of the pulsar at any time $t \geq t_1$. The rate of change of the period P is

$$\dot{P} = -\frac{2\pi}{\Omega^2}\frac{d\Omega}{d\tau} = \frac{\pi}{\Omega_0\tau_p}[1 + (\tau - \tau_0)/\tau_p]^{-1/2}, \tag{73}$$

and the present rate of change is

$$\dot{P}_0 = \pi/\Omega_0\tau_p = P_0/2\tau_p . \tag{74}$$

To order of magnitude, $M_\perp \approx BR^3$, $I_0 \approx MR^2$, where B is the magnetic field strength of the pulsar. If we take the typical values $B \approx 10^{12}$ gauss, $R \approx 10^6$ cm, $M \approx 2 \times 10^{33}$ g, we get $\tau_p = 4 \times 10^{16}\Omega_0^{-2}$ s. When this expression for τ_p is used in (74), we get good agreement with the results of most pulsars. Experimentally, $\dot{P} \sim P^{2-n}$, where $n = 2.33$ to 3 (Ruderman 1972).

d) *M-G-R Relation*

In order to apply the LNH to a neutron star, we must know the *M-G-R* relation. The effects of the LNH on M and G are of course known. The *M-G-R* relation can be easily obtained by making a homologous transformation of the relevant equations of stellar structure if we assume that the neutron star is cold (zero temperature) and is not generating energy. The relevant equations are then the nonrelativistic equation of hydrostatic equilibrium,

$$\frac{dp_r}{dr} = -\frac{G\rho_r M_r}{r^2}, \tag{75}$$

and the differential equation for mass,

$$\frac{dM_r}{dr} = 4\pi r^2 \rho_r \,. \tag{76}$$

In the above two equations p_r, ρ_r, and M_r are the pressure, density, and total mass enclosed, respectively, at the distance r from the center of the star. In addition, we shall assume a polytropic equation of state

$$p = k\rho^{\gamma} \,, \tag{77}$$

where γ is related to the polytropic index n by

$$\gamma = 1 + 1/n \,. \tag{78}$$

If we now consider the homologous transformations $G \to G^* = gG$, $M_r \to M_r^* = qM_r$, $\rho_r \to \rho_r^* = j\rho_r$, and $r \to r^* = xr$, equations (75)–(77) give

$$k\frac{j^{\gamma}}{x}\frac{d\rho_r{}^{\gamma}}{dr} = -\frac{jgq}{x^2}\frac{G\rho_r M_r}{r^2} \,, \tag{79}$$

$$\frac{q}{x}\frac{dM_r}{dr} = x^2 j 4\pi r^2 \rho_r \,. \tag{80}$$

If the above equations must remain invariant, the products of new factors on both sides of the equations must be equal. Equation (79) therefore gives

$$j^{\gamma}x = jgq \,,$$

while (80) gives

$$q = x^3 j \,.$$

Eliminating j gives

$$gx^{3\gamma-4}q^{2-\gamma} = 1$$

or

$$GM^{2-\gamma}R^{3\gamma-4} = \text{const.} = \delta \,. \tag{81}$$

The zero-temperature equation of state for a perfect nonrelativistic gas has $\gamma = 5/3$, but we shall use the more general relation (81).

e) *Effects of LNH*

We are now in a position to write down the time behavior of the moment of inertia of a cold star with a polytropic equation of state within the LNH framework. We write the mass of the star as

$$M(t) = M_0(t/t_0)^2 \,, \tag{82}$$

where M_0 is the mass at cosmic time t_0, which we take to be the present. Similarly, we write the gravitational constant as

$$G(t) = G_0(t_0/t) \,. \tag{83}$$

The moment of inertia is given by $I(t) = \beta MR^2$, where β is some constant. If we now use the relations (81), (82), and (83), we obtain for the moment of inertia

$$I(t) = I_0(t/t_0)^{2(5\gamma-7)/(3\gamma-4)} \,, \tag{84}$$

where I_0 is the present moment of inertia given by

$$I_0 = \beta\delta^{2/(3\gamma-4)}G_0{}^{2/(4-3\gamma)}M_0{}^{(5\gamma-8)/(3\gamma-4)} \,. \tag{85}$$

However, before we can write down the analog of (68), we must take into account the fact that kinetic energy is not conserved in the LNH because of the continuous creation of matter. In order to see how the kinetic energy is affected, let us consider a body of mass m moving with a velocity v. Since velocity is constant in the LNH while mass increases at t^2, kinetic energy also increases as t^2, i.e., $K = K_0(t/t_0)^2$, so that the rate of change of kinetic energy must be

$$\frac{dK}{dt} = 2K_0 t/t_0^2 . \tag{86}$$

The analog of equation (68) for the rate of change of the rotational kinetic energy of a pulsar in the LNH is therefore

$$\frac{d}{dt}\left[\tfrac{1}{2}I_0\left(\frac{t}{t_0}\right)^\alpha \Omega^2\right] = -\frac{\lambda}{2}\Omega^4 + 2K_0 t/t_0^2 , \tag{87}$$

where

$$K_0 = \tfrac{1}{2}I_0\Omega_0^2 , \qquad \tfrac{1}{2}\alpha \equiv \frac{5\gamma - 7}{3\gamma - 4} . \tag{88}$$

With the transformation

$$\Omega^2 = qt^\alpha \dot{h}/h , \qquad q \equiv \frac{I_0}{\lambda t_0^\alpha} . \tag{89}$$

Equation (87) becomes

$$\ddot{h} + \frac{2\alpha}{t}\dot{h} - pt^{1-2\alpha}h = 0 \tag{90}$$

with

$$p = 4K_0\lambda t_0^{2\alpha-2}I_0^{-2} = 4K_0 t_0^{\alpha-2}I_0^{-1}q^{-1} .$$

The quantity of interest to us is $\dot{\Omega}/\Omega$ or $\dot{P}/P$. Using (89) and (90), we obtain

$$\frac{\dot{\Omega}}{\Omega} = -\frac{\alpha}{2t} + \tfrac{1}{2}pq\frac{t^{1-\alpha}}{\Omega^2} - \frac{\Omega^2}{2qt^\alpha} , \qquad \frac{\dot{P}}{P} = \frac{\alpha}{2t} - \tfrac{1}{2}pq\frac{t^{1-\alpha}}{\Omega^2} + \frac{\Omega^2}{2qt^\alpha} . \tag{91}$$

The present value of $\dot{P}/P$ is easily evaluated to be

$$\left(\frac{\dot{P}}{P}\right)_0 = (\tfrac{1}{2}\alpha - 1)\frac{1}{t_0} + \frac{1}{2\tau_p} , \qquad \left(\frac{\dot{P}}{P}\right)_0 = \frac{2\gamma - 3}{3\gamma - 4}\left(-\frac{\dot{G}}{G}\right)_0 + \frac{1}{2\tau_p} . \tag{92}$$

Recently Heintzmann and Hillebrandt (1975) have derived an equation for $\dot{P}/P$ based on the variation of G alone. However, in the light of the criticism of Gamow and Teller discussed before, such a model is unphysical.

The pertinent observational data for the pulsar JP 1953 are

$$P = 0.426676 \text{ s} , \qquad \dot{P} = 0.34 \times 10^{-17} ,$$

$$\dot{P}/P = 25.14735 \times 10^{-11} \text{ yr}^{-1} , \qquad P/\dot{P} = 3.98 \times 10^9 \text{ yr} , \tag{93}$$

whereas the latest determinations of $\dot{G}/G$ are (Shapiro *et al.* 1971)

$$\left|\frac{\dot{G}}{G}\right| \equiv 4 \times 10^{-10} \text{ yr}^{-1} \tag{94a}$$

and (Van Flandern 1975 and private communication)

$$-\frac{\dot{G}}{G} = (9 \pm 4) \times 10^{-11} \text{ yr}^{-1} . \tag{94b}$$

If $B \ll 10^{11}$ gauss, i.e., if the last term in (92) is too small to fit the data, Dirac's theory can constitute an alternative explanation, thus providing a lower limit for $\dot{P}/P$.

Evidently the rate of change of period for JP 1953 could be explained if we assume that we are dealing with a typical white dwarf instead of a neutron star with an abnormally low magnetic field. (This model seems to have been first proposed by F. Drake.) If we use for the mass of the white dwarf $M \approx M_\odot$, a radius $\sim 2.5 \times 10^8$ cm, and a typical magnetic field $\sim 10^6$ gauss, then (92) gives for the rate of change of period $\dot{P} \approx 4 \times 10^{-18}$ s s^{-1}, which agrees quite well with the experimental value of 3.4×10^{-18} s s^{-1}. However, the period of this pulsar, 0.43 s, is dangerously close to the critical period for centrifugal disruption of a white dwarf.

f) White Dwarf Luminosity

Stothers (1975) has recently shown that the luminosity of a typical white dwarf due to the effects of the LNH *alone* should be

$$L_{\mathrm{WD}} = \frac{d}{dt}\left(\frac{GM^2}{R}\right) \approx 0.2\, L_\odot\,, \tag{95}$$

where $L_\odot$ is the present solar luminosity. This result is in conflict with the observational evidence, $L_{\mathrm{WD}} \approx 10^{-2}$–$10^{-3}\, L_\odot$. Stothers assumes that if the LNH is correct, then the additional luminosity must have been radiated in some undetected form or must have been exactly used up in the creation of new matter inside the white dwarf.

The same argument leads to a luminosity of a typical neutron star,

$$L_{\mathrm{NS}} \approx 10^2\, L_\odot\,, \tag{96}$$

with a corresponding effective surface temperature of $\approx 10^6$ K which is comparable to the expected temperature of a typical neutron star in the conventional theory (Tsuruta *et al.* 1972).

V. THEORY OF DICKE AND CARTER

Having presented some of the results of Dirac's LNH, we would like to mention different lines of argument aimed at explaining the existence of the large numbers (1), (3), and (4) (Dicke 1961; Carter 1974). In this theory, the large dimensionless numbers are not connected to each other through their relations to the cosmic time t as in Dirac's LNH, but are related because we, as observers, occupy a privileged position in time. Dicke argues that life could not have existed until the heavier elements had been synthesized in stars, and the time required to do so is of the order of the maximum age of a star $t_{\max}$ capable of producing energy through nuclear reactions. When the mass and luminosity of such a star are expressed (to orders of magnitude) in terms of the fundamental constants, then $t_{\max}$ turns out to be

$$t_{\max} \approx \left(\frac{m_p}{m_e}\right)^{1/2}\left(\frac{e^2}{\hbar c}\right)\left(\frac{e^2}{Gm_p m_e}\right)\left(\frac{e^2}{m_e c^3}\right). \tag{97}$$

Since the Hubble parameter H is of the order of ${t_{\max}}^{-1}$ for an evolutionary cosmology, we get from (97) the relation

$$m_e c^3/e^2 H \approx e^2/Gm_p m_e$$

after dropping factors of order unity. This is exactly the relation we get from (1) and (3). It should be noted that this relation holds only if $t_{\max} \approx t_0$, the present epoch. Far into the future $t_{\max}$ will be much less than the corresponding epoch, so (97) will not hold. Hence our rather privileged position in time is responsible for (97). In order to explain the result (16), i.e., $GM_u H/c^3 \approx 1$, which results naturally from the LNH, Dicke must invoke Mach's principle which requires just such a relationship (Sciama 1953).

VI. CONCLUSIONS

Dirac's LNH provides an attractive explanation for the large dimensionless numbers and makes some definite predictions, such as the value of q_0 and $(\dot{G}/G)_0$, that are in agreement with experiments. Some of its other predictions, such as the steepening of the (log N, log S)-curve for radio source counts and the rate of change of pulsar periods, are consistent with observation.

However, difficulties of the theory still exist. If the cosmic microwave background radiation does have a black-body spectrum, it would have to be a coincidence according to the LNH, unless some other method of producing this spectrum exists. Also, some mechanism must be invoked to explain the unexpectedly low luminosities of white dwarfs, if the LNH is to become a viable theory.

Finally we would like to note that one of the authors (V. C.) in collaboration with P. J. Adams, H. S. Hsieh, and E. Tsiang (*Phys. Rev. D.*, submitted 1976) has constructed a gauge invariant theory of gravitation in which the large number hypothesis finds a more natural explanation.

One of the authors (V. C.) would like to express his gratitude to Professor P. A. M. Dirac for his detailed explanation of the theory and for most valuable and enlightening correspondence.

REFERENCES

Bahcall, J. N., and Schmidt, M. 1967, *Phys. Rev. Letters*, **19**, 129.
Carter, B. 1974, in *IAU Symposium No. 63*, ed. M. S. Longair (Dordrecht: Reidel), p. 291.
Chin, C. W., and Stothers, R. 1975, *Nature*, **254**, 206.
Counselman, C. C., and Shapiro, I. I. 1968, *Science*, **162**, 352.
Dicke, R. H. 1961, *Nature*, **192**, 440.
Dirac, P. A. M. 1937, *Nature*, **139**, 323.
———. 1973*a*, *Pont. Acad. Commentarii*, **11**, 46, 1.
———. 1973*b*, *Proc. Roy. Soc. London*, **A333**, 403.
———. 1973*c*, in *The Physicist's Conception of Nature*, ed. J. Mehra (Dordrecht: Reidel), p. 45.
———. 1974, *Proc. Roy. Soc. London*, **A333**, 439.
———. 1975, in *Theories and Experiments in High-Energy Physics*, B. Kursunoglu *et al.* (New York: Plenum).
Gamow, G. 1967, *Proc. Nat. Acad. Sci.*, **57**, 187.
Gott, J. R., Gunn, J. E., Schramm, D. N., and Tinsley, B. M. 1974, *Ap. J.*, **194**, 543.
Heintzmann, H., and Hillebrandt, W. 1975, *Phys. Letters*, **54A**, 349.
Longair, M. S. 1971, *Rept. Progr. Phys.*, **34**, 1125.
Narlikar, J. V. 1974, *Nature*, **247**, 99.
Pochoda, P., and Schwarzschild, M. 1964, *Ap. J.*, **139**, 587.
Richards, D. W., Rankin, J. M., and Zeissig, G. A. 1974, *Nature*, **251**, 37.
Ruderman, M. 1972, *Ann. Rev. Astr. and Ap.*, **10**, 427.
Sandage, A. 1961, *Ap. J.*, **133**, 355.
———. 1968, *Observatory*, **88**, 91.
———. 1972, *Ap. J.*, **173**, 485.
———. 1972*a*, *Ap. J.*, **178**, 1.
———. 1972*b*, *Ap. J.*, **178**, 25.
———. 1974, *Ap. J.*, **194**, 223.
Sandage, A., and Tammann, G. 1975, *Ap. J.*, **196**, 313.
Sciama, D. W. 1953, *M.N.R.A.S.*, **113**, 34.
Shapiro, I. I., Smith, W. B., Ash, M. B., Ingalls, R. P., and Pettengill, G. H. 1971, *Phys. Rev. Letters*, **26**, 27.
Stothers, R. 1975, Institute for Space Studies preprint.
Teller, E. 1948, *Phys. Rev.*, **73**, 801.
Tsuruta, S., Canuto, V., Lodenquai, J., and Ruderman, M. 1972, *Ap. J.*, **176**, 739.
Van Flandern, T. C. 1975, *M.N.R.A.S.*, **170**, 333.

Y. M. CANUTO: Institute for Space Studies, NASA 2880 Broadway, NY 10025

J. LODENQUAI: Phys. Dept., U.W.I., Mona, Kingston, Jamaica